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Active Vibration Control of Flexible Structures with Acceleration Feedback

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Introduction

THE vibration suppression problem is an important aspect of the overall control problem of flexible structures such as rotary and fixed-wing aircraft, as well as spacecraft. The amplitude and duration of vibration must be controlled so that the structures do not experience performance degradation or structural damage. One of the techniques that can be used for the control of flexible structures is collocated control, in which sensors and actuators are placed in the same location. Collocated control appears to be suitable for structures with distributed sensors and actuators.

Collocated velocity feedback has been studied in the past[1,2] in conjunction with the control of large flexible space structures. Collocated velocity feedback is unconditionally stable in the absence of actuator dynamics. However, in the presence of actuator dynamics, the stability condition is dependent on structural damping and frequency.[3] The combined structure and controller system will be stable if the actuator dynamics are sufficiently fast. Unfortunately, flexible structures have infinite bandwidth, whereas all actuators have finite bandwidth. Accordingly, the system stability is not guaranteed and instability could result.[3,4] The inherent lack of overall system stability renders the velocity feedback scheme less attractive.

Collocated positive position feedback overcomes the shortcomings of velocity feedback.[3] Although it is not unconditionally stable, the stability condition of positive position feedback is independent of uncertain structural damping and the actuator dynamics. Fanson and Caughey[4] conducted analytical and experimental work using a cantilever beam as an example. A large increase in damping was achieved, demonstrating the feasibility of using position feedback as a strategy to control the vibration of flexible structures. However, in positive position feedback, the stability condition may be stringent in that the magnitude of the controller gain is limited by the lowest frequencies of the structure. This may place a limit on the system damping and the overall system performance that can be achieved through positive position feedback control.

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Alternately, we investigate in this Note collocated acceleration feedback including finite actuator dynamics. In the next section, the stability of acceleration feedback control for flexible structures is investigated. It will be shown that when the acceleration is fed back in the collocated control, the combined structure-controller system is unconditionally stable and independent of the damping and natural frequencies of the structure to be controlled. Subsequently, vibration control of a cantilever beam is used to demonstrate the effectiveness of the control scheme.

A more general case of the control law considered in this paper has been investigated in Refs. 5–7 that covers both analytical and experimental aspects.

Acceleration Feedback Control

An implementation of collocated acceleration feedback control for flexible structures can be written in matrix form as

Structure:

$$\ddot{\xi} + D\dot{\xi} + \Omega\xi = -P^T G\eta \tag{1}$$

Controller:

$$\ddot{\eta} + D_a\dot{\eta} + \Omega_a\eta = \Omega_a P\ddot{\xi} \tag{2}$$

where the vectors and matrices are defined as follows:

$\xi \equiv$ structure modal vector of length n_m
$\eta \equiv$ controller state vector of length n_a
$G \equiv$ gain matrix:diag $n_a \times n_a$, $g_i > 0$ for all i

$$= \begin{bmatrix} g_1 & & 0 \\ & \ddots & \\ 0 & & g_{n_a} \end{bmatrix}$$

$P \equiv$ participation matrix:$n_a \times n_m$
$\Omega \equiv$ structural frequency matrix:diag $n_m \times n_m$

$$= \begin{bmatrix} \omega_1^2 & & 0 \\ & \ddots & \\ 0 & & \omega_{n_m}^2 \end{bmatrix}$$

$\Omega_a \equiv$ controller frequency matrix:diag $n_a \times n_a$

$$= \begin{bmatrix} \omega_{a_1}^2 & & 0 \\ & \ddots & \\ 0 & & \omega_{a_{n_a}}^2 \end{bmatrix}$$

$D \equiv$ structural damping matrix:diag $n_m \times n_m$

$$= \begin{bmatrix} 2\zeta_1\omega_1 & & 0 \\ & \ddots & \\ 0 & & 2\zeta_{n_m}\omega_{n_m} \end{bmatrix}$$

$D_a \equiv$ controller damping matrix:diag $n_a \times n_a$

$$= \begin{bmatrix} 2\zeta_{a_1}\omega_{a_1} & & 0 \\ & \ddots & \\ 0 & & 2\zeta_{a_{n_a}}\omega_{a_{n_a}} \end{bmatrix}$$

Then, we can prove the following theorem for the stability condition of the system.

Theorem: The combined structure and controller dynamics of Eqs. (1) and (2) are unconditionally Lyapunov asymptotically stable (LAS), regardless of the damping and natural frequencies of the structure.

Proof: By differentiating Eq. (1) with respect to time and introducing a new variable

$$\xi_n \equiv \dot{\xi} \tag{3}$$

Eqs. (1) and (2) can be written as

$$\ddot{\xi}_n + D\dot{\xi}_n + \Omega\xi_n = -P^T G\dot{\eta} \tag{4}$$

$$\ddot{\eta} + D_a\dot{\eta} + \Omega_a\eta = \Omega_a P\dot{\xi}_n \tag{5}$$

To symmetrize the foregoing equations, we use the following nonsingular transformation:

$$\eta = G^{-T/2}\Omega_a^{T/2}\psi \tag{6}$$

where $\Omega_a^{T/2} = [\Omega_a^{1/2}]^T$ and $G^{-T/2} = [G^{-1/2}]^T$.
Substituting Eq. (6) into Eqs. (4) and (5) and premultiplying Eq. (5) by $\Omega_a^{-1/2}G^{1/2}$, we obtain the following pair of equations:

$$\ddot{\xi}_n + D\dot{\xi}_n + \Omega\xi_n = -P^TG^{T/2}\Omega_a^{T/2}\dot{\psi} \tag{7}$$

$$\ddot{\psi} + D_a\dot{\psi} + \Omega_a\psi = \Omega_a^{1/2}G^{1/2}P\dot{\xi}_n \tag{8}$$

Equations (7) and (8) can be written in matrix form as

$$\begin{bmatrix} \ddot{\xi}_n \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} D & P^TG^{T/2}\Omega_a^{T/2} \\ -\Omega_a^{1/2}G^{1/2}P & D_a \end{bmatrix}\begin{bmatrix} \dot{\xi}_n \\ \dot{\psi} \end{bmatrix}$$

$$+ \begin{bmatrix} \Omega & 0 \\ 0 & \Omega_a \end{bmatrix}\begin{bmatrix} \xi_n \\ \psi \end{bmatrix} = 0 \tag{9}$$

Now we may define the Lyapunov function V as

$$V = \frac{1}{2}[\dot{\xi}_n^T\dot{\psi}^T]\begin{bmatrix} \dot{\xi}_n \\ \dot{\psi} \end{bmatrix} + \frac{1}{2}[\xi_n^T\psi^T]\begin{bmatrix} \Omega & 0 \\ 0 & \Omega_a \end{bmatrix}\begin{bmatrix} \xi_n \\ \psi \end{bmatrix} \tag{10}$$

Then $V > 0$ for all nontrivial ξ_n, ψ, $\dot{\xi}_n$, $\dot{\psi}$. Differentiating V with respect to time yields

$$\dot{V} = [\dot{\xi}_n^T\dot{\psi}^T]\begin{bmatrix} \ddot{\xi}_n \\ \ddot{\psi} \end{bmatrix} + [\xi_n^T\psi^T]\begin{bmatrix} \Omega & 0 \\ 0 & \Omega_a \end{bmatrix}\begin{bmatrix} \dot{\xi}_n \\ \dot{\psi} \end{bmatrix} \tag{11}$$

Substituting Eq. (9) into Eq. (11), we get

$$\dot{V} = [\dot{\xi}_n^T\dot{\psi}^T]\begin{bmatrix} -D & -P^TG^{T/2}\Omega_a^{T/2} \\ \Omega_a^{1/2}G^{1/2}P & -D_a \end{bmatrix}\begin{bmatrix} \dot{\xi}_n \\ \dot{\psi} \end{bmatrix} \tag{12}$$

or

$$\dot{V} = [\dot{\xi}_n^T\dot{\psi}^T]\begin{bmatrix} -D & 0 \\ 0 & -D_a \end{bmatrix}\begin{bmatrix} \dot{\xi}_n \\ \dot{\psi} \end{bmatrix}$$

$$+ [\dot{\xi}_n^T\dot{\psi}^T]\begin{bmatrix} 0 & -P^TG^{T/2}\Omega_a^{T/2} \\ \Omega_a^{1/2}G^{1/2}P & 0 \end{bmatrix}\begin{bmatrix} \dot{\xi}_n \\ \dot{\psi} \end{bmatrix} \tag{13}$$

The second term of Eq. (13) is zero. Thus,

$$\dot{V} = -\dot{\xi}_n^TD\dot{\xi}_n - \dot{\psi}^TD_a\dot{\psi} \tag{14}$$

Since both D and D_a are positive definite

$$\dot{V} < 0 \tag{15}$$

for nontrivial $\dot{\xi}_n$ and $\dot{\psi}$. Therefore, the system represented by Eqs. (1) and (2) is LAS by Lyapunov's direct method.[8]

Examples

To demonstrate the effectiveness of the acceleration feedback control scheme, vibration control of a cantilever aluminum beam was chosen as an example problem. This example is similar to that used in Ref. 9, except that the length dimension was scaled by a factor of 2 to lower the natural frequencies. Further, the beam thickness was adjusted to give a thickness-to-width ratio of 10. In the present study, we assume that there exists a means to obtain necessary measurements without considering the details. This assumption is adequate for the present analytical investigation. The beam was modeled by 10 three-node finite elements based on the Timoshenko theory that allows transverse shear deformation. In the present exercise, only the first four modes were retained. A damping ratio of $\zeta = 0.001$ was assumed for all modes.

Initially, the first mode was controlled by one actuator/sensor pair located near the fixed end of the beam. The controller design frequency and damping ratio were set as 100 rad/s and 70%. When the gain g_1 was set as 1.07×10^5, the maximum damping for the first mode was obtained. A summary of open- and closed-loop frequency ω (rad/s) and damping ζ is contained in Table 1.

Table 1 One-mode control performance

Structure mode	Open-loop ω	Open-loop ζ, %	Closed-loop ω	Closed-loop ζ, %
1	32.026	0.10	22.291	10.3
2	192.85	0.10	255.08	2.03
3	536.57	0.10	720.48	1.09
4	1055.1	0.10	1806.1	2.85

Subsequently, two actuator/sensor pairs were used to control the first two modes. The second controller was given a design frequency of 250 rad/s and damping ratio of 70%. When the gain of the first controller was set as $g_1 = 1.05 \times 10^5$ and that of the second controller as $g_2 = 1.90 \times 10^4$, a favorable result was obtained. A summary of open- and closed-loop performance is contained in Table 2. In comparison with the preceding case, we observe an increase in the damping ratio of the second mode due to the addition of a second actuator/sensor pair.

Table 2 Two-mode control performance

Structure mode	Open-loop ω	Open-loop ζ, %	Closed-loop ω	Closed-loop ζ, %
1	32.026	0.10	22.427	10.2
2	192.85	0.10	178.58	34.7
3	536.57	0.10	752.18	1.31
4	1055.1	0.10	2380.8	4.48

Conclusions

The global stability condition has been examined for acceleration feedback control. The combined structure-controller system is unconditionally stable, regardless of the natural frequencies and damping of the structure being controlled. Accordingly, the system is stable even in the presence of unmodeled or inaccurately modeled modes and damping. The results of the vibration control exercise on a simple cantilever beam demonstrate the effectiveness of the control scheme. The present Note did not consider the actual implementation aspects of this control scheme. These issues, as well as experimental verification, will be the subject of future investigations.

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